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Alan Turing: Virtuosity and visualisation

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Alan Turing (1912–1954) has been increasingly recognised as an important mathematician who, despite his short life, developed mathematical ideas that today have led to foundational aspects of computer science, especially with respect to computability, artificial intelligence and morphogenesis (the growth of biological patterns, important in mathematical biology). Some of Turing's mathematics and related ideas can be visualised in interesting and even artistic ways, aided using software. In addition, a significant corpus of the historical documentation on Turing can now be accessed online as a number of major archives have digitised material related to Turing. This paper introduces some of the major scientific work of Turing and ways in which it can be visualised, often artistically. Turing's fame has, especially since his centenary in 2012, reached a level where he has had a cultural influence on the arts in general. Although the story of Turing can be seen as one of tragedy, with his life cut short, from a historical viewpoint Turing's contribution to humankind has been triumphant.

Artificial Intelligence. Digital art. History of mathematics. Morphogenesis. Turing Machines. Visualisation.

1. BACKGROUND AND INTRODUCTION

Alan Mathison Turing, OBE, FRS (23 June 1912 – 7 June 1954) was a British mathematician and codebreaker (Newman 1955, Hodges 1983/2012). He is considered by many to be the founder or father of computer science (Bowen 2012, 2013). A number of meetings celebrating the centenary of his birth in 2012 were held in the United Kingdom and even internationally, including at Bletchley Park (the site of his wartime codebreaking work), Cambridge, Manchester, Oxford, etc. He has become increasingly embedded in the public consciousness, especially the UK, but also beyond.

In this paper, we explore various aspects of Turing', who died when only aged 41, just 16 days short of his 42nd birthday: in particular his scientific contributions and associated visualisations of these. Despite his short life, Turing made very important contributions to the fields of mathematics, philosophy, and perhaps most importantly computer science, producing an eponymous theory of computation machines, foundational in theoretical computer science.

In Section 2, Turing's major scientific contributions are considered in overview, including the concept of a Turing Machine, with visualisations related to of some of Turing's ideas. Section 3 presents some Turing-related artwork, leading to a new book cover. In Section 4, major archives relating to Turing are presented, including his academic output. Finally, Section 5 provides an epitaph, together with noting some of Turing's cultural influence in the arts.

2. MATHEMATICS AND COMPUTER SCIENCE

> "Mathematical reasoning may be regarded rather schematically as the exercise of a combination of two facilities, which we may call intuition and ingenuity."
>
> – Alan Turing (1938)

The contribution of Alan Turing to mathematics and computer science is exceptional, especially considering his early death at 41. He is an example of the widely held notion that a mathematician produces their best work when young (Bowen and Giannini 2015), typically in their 20s, when Turing developed his theoretical concept of a universal computing machine. Creative breakthroughs need years of commitment and complete dedication, often with a decade of lead-up time to produce a work of genius (Robinson 2010). Turing's first important

work of influence was published when he was aged 24 and he already had an understanding of the work of Einstein in his teens through independent study (Hodges 1983, p. 542)

2.1 Turing Machines

Turing's first major paper introduced what was later to become known as a Universal Turing Machine (Turing 1936). This theoretical idea of a simple computational device can be used to explore what is computable. It consists of a machine with an infinite tape of symbols and the ability to read from and write to the tape at the current position, and to move the tape left or right. It turns out that it is equivalent to many other computational models and devices, including (if unrestricted in size) the modern digital computer.

The execution of a Turing Machine can be visualised using software, for example Mathematica, which provides mathematical support as well as visualisation facilities. For example, the input to visualise the evolution of a 2-state, 2-colour (or symbol) Turing machine number "2506" running for 50 steps is as follows (Wolfram 2016):

```
ArrayPlot[Function[u, MapAt[Red &,
u[[2]], u[[1, 2]]]]/@TuringMachine
[2506,{1, {{}, 0}, 50]]
```

This produces visualisation output as in Figure 1.

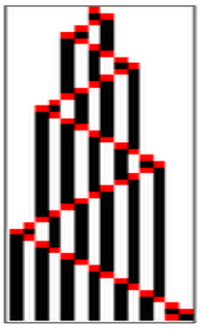

Figure 1: *Turing Machine execution visualisation (Wolfram 2016)*

John Conway devised the "Game of Life" in 1970, another computational model, computationally equivalent to a Turing Machine. This has simple rules for deciding whether cells in a potentially infinite two-dimensional grid live or die depending on the number of neighbours. Living cells with two or three living neighbours live on, otherwise they die in the next generation. Dead cells with exactly three living neighbours become alive in the next generation, a form of reproduction. The grid is seeded with a pattern and then the rules are applied potentially ad infinitum with successive generations. This can generate very complex and sometimes aesthetic patterns. It is even possible to simulate a Turing Machine visually (Rendell 2010), as shown in Figure 2, and the Game of Life itself (Bradbury 2012) in a beautiful even artistic way, visually demonstrating the universality of Conway's Game of Life in a compelling and dramatic way. The last image in Figure 2 shows the computation part of the machine in the rectangular box and the tape running diagonally on the top right.

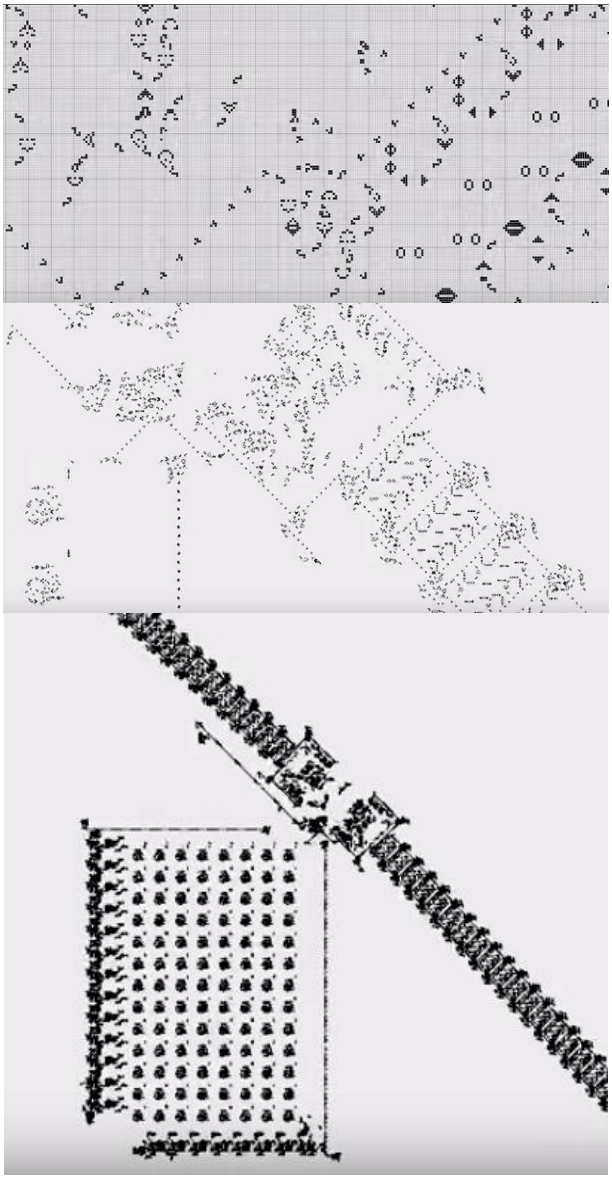

Figure 2: *A Turing Machine visualisation in Conway's Game of Life at different magnifications (Rendell 2010)*

2.2 Artificial Intelligence

Turing (1950) predicted Artificial Intelligence (AI), which he termed "machine intelligence". He wrote:

> "I believe that at the end of the [20th] century, the use of words and general educated opinion will have altered so much that one will be able to speak of machines thinking without expecting to be contradicted."
>
> – Alan Turing (1950)

He devised the Turing Test to determine if a computer can mimic a human with textual interaction sufficiently well to fool a real human most of the time.

Although there have been claims to have succeeded with this test, this is still an active area of research, with good progress in machine learning more recently.

Conway's Game of Life, as previously mentioned, although based on very simple rules, can exhibit AI-like properties (Degirmentas 2015). Just as a Turing Machine can be created in the Game of Life, as illustrated in Figure 2, more artistic complex patterns can be generated. For example, see Ascani (2011), as show in Figure 3 below.

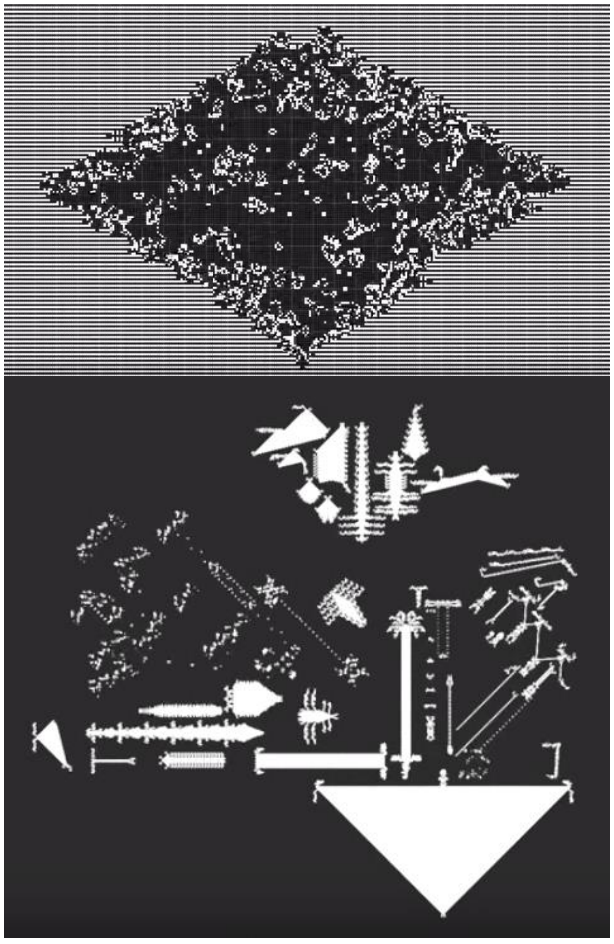

Figure 3: *Visualisations of artistic patterns in Conway's Game of Life (Ascani 2011)*

2.3 Morphogenesis

Towards the end of Turing's life, he investigated morphogenesis, the way in which natural patterns in biology develop through chemical means (Turing 1952). He used simple equations to demonstrate the principle that complex and unpredictable patterns can be generated using very little mathematics. One of the few (six) references included by Turing was Thompson (1942), the 2nd edition of *On Growth and Form*, originally published in 1917. The author, Sir D'Arcy Wentworth Thompson FRS (1860–1948), was a Scottish biologist and mathematician who was a pioneer of mathematical biology. Although a scientist at heart, D'Arcy Thompson was inspired by artists such as Albrecht Dürer (1471–1528).

Bernard Richards, Turing's last Master's student at the University of Manchester from 1953, worked on morphogenesis for his thesis, including a number if illustrations of spherical biological forms and mathematical plots generating similar shapes (Richards 1954). Figure 4 and Figure 5 illustrate *Circopus sexfurcus*, a type of small *Radiolaria* unicellular protozoa, and a mathematical plot matching with this shape, using morphogenesis equations on a sphere, as presented in the thesis.

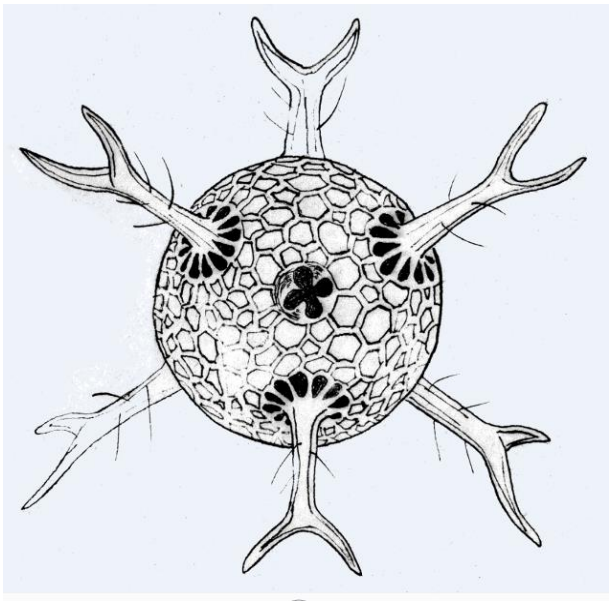

Figure 4: *Circopus sexfurcus with six spines at 90° apart (Richards 1954), derived from Haeckel (1873–76) (Included with permission of Bernard Richards)*

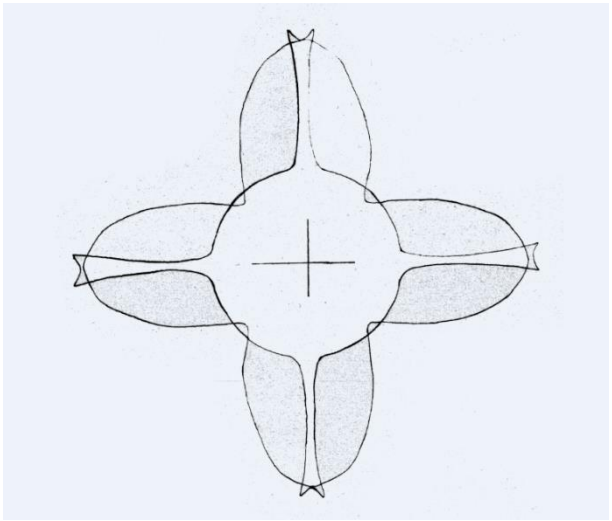

Figure 5: *Computer solution superimposed on Circopus sexfurcus (Richards 1954) (Included with permission of Bernard Richards)*

The equations are relatively simple (Richards 2005–6):

$$\frac{dU}{dt} = \Phi(\nabla^2)U + GU^2 - HUV,$$

$$V = \overline{U}^2$$

where U is the vector from the centre to the surface of the membrane. However the algebraic solution is much more complicated, running to around 30 pages in Richards (1954).

Morphogenesis can be visualised in interesting ways, For example, Figure 6 illustrates an example of visualised morphogenesis on the front cover of *Science*, a leading international journal. This was part of an annual visualisation challenge in 2009 (Nesbit and Bradford 2010). The cover artwork, "*Branching Morphogenisis*", by Jenny E. Sabin *et al*., illustrates an installation created using over 75,000 cable zip ties, representing predicted forces produced by cells in a human lung, forming networks over time (Science 2010)

Figure 6: *Front cover "Branching Morphogenesis" visualisation (Science 2010)*

Morphogenesis is now a very active research area and Turing (1952) is his most cited paper, even though it was published only two years before his death. Who knows what more Turing could have achieved had his life not been cut short since he was still at the height of production of his influential ideas.

As well as being foundational in mathematical biology, morphogenesis is also influential in art. For example, Andy Lomas (2016) is using mathematics to generate complex artificial life forms for artistic purposes, in 2D, 3D and moving forms, as exhibited at Watermans (Kew, west London) in the exhibition *Morphogenetic Creations* during 2016 (Watermans 2016). See Figure 7 and Figure 8. The exhibition was accompanied by an interesting historical talk, relating science and art in the context of morphogenesis in general as well as in the context of D'Arcy Thompson in particular (Jarron 2016).

Figure 7: *Andy Lomas introducing his exhibition "Morphogenetic Creations" at Watermans, London, 15 June 2016 (Photograph by J. P. Bowen, 2016)*

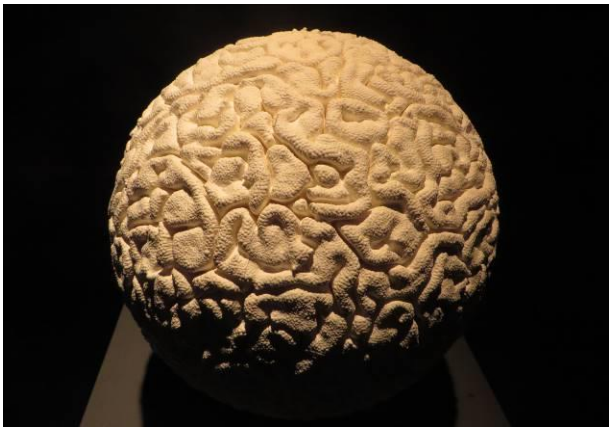

Figure 8: *Morphogenetic sculpture by Andy Lomas (Photograph by J. P. Bowen, 2016)*

3. ARTWORK AND COVERS

Science can produce interesting visualisations and journals often use the fact to produce striking and colourful cover artwork (Bowen *et al*. 2016, see also Figure 6). With respect to Turing, there are few photographs, all are monochrome, and there is no known film footage or audio recording of him, despite living in the 20th century. Figure 9 shows the classic well-known black and white photograph of Turing, with some image sharpening.

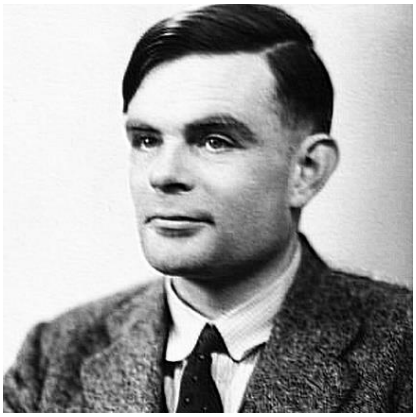

Figure 9: *Classic Alan Turing photograph (with sharpening)*

Figure 10 shows a processed version of this photograph, using some standard free image software, IrfanView (http://www.irfanview.com), utilising solarisation and edge detection effects, generating a flash of lightening on the right over Turing's shoulder, potentially a strong enough image for use on a front cover of a publication.

Figure 10: *Turing image using solarisation and edge detection effects, derived from Figure 9 (By J. P. Bowen, 2016, using IrfanView)*

The *Warholize.me* website, which has since become *The Pictomizer* (http://pictomizer.com), allowed the creation of Warhol-like images from a source photograph. Andy Warhol used his screen-print approach for a number of famous people, including Jackie Kennedy, Marilyn Monroe, and even Chairman Mao, but never Turing, who was not very well-known at the time, especially in the US.

Figure 11: *Initial mock-up artwork for The Turing Guide (Copeland et al. 2017), inspired by Andy Warhol (By J. P. Bowen, 2012, using http://warholize.me)*

It is interesting to speculate whether if Andy Warhol had been active today, he might have chosen Turing as subject. With this in mind, and given the striking and colourful nature of Warhol's images, it was decided that it could make a good book cover for *The Turing Guide*, a new book on Turing, with its genesis in 2012 centenary meetings at Bletchley Park, Cambridge, and Oxford (Copeland *et al*. 2017). An initial mock-up is shown in Figure 11. Subsequently this idea has been worked up into a more professional version for the actual front cover.

4. ARCHIVES AND SCHOLARSHIP

Archival material relating to Turing is increasingly accessible online, as demand for digital content grows (Bowen and Giannini 2014). Archival documents can help to visualise history (Bowen and Giannini 2015). There are a number of excellent online archives relating to Turing, including:

- *The Alan Turing Year* – started to celebrate Turing's life and work for his 2012 centenary year (http://www.turingcentenary.eu)
- *Alan Turing: The Enigma* – by Andrew Hodges, Turing's biographer (http://www.turing.org.uk)
- *The Turing Digital Archive* – nearly 3,000 images from King's College, Cambridge, Turing's college (http://www.turingarchive.org)
- *The Turing Archive for the History of Computing* – digital facsimiles by Jack Copeland and Diane Proudfoot (http://www.alanturing.net)

With respect to visualisation, in many academic disciplines, family trees of academic supervisors (normally for PhD studies) are a popular pastime, enabling the search for famous supervisory "ancestors". The online Mathematics Genealogy Project (http://genealogy.math.ndsu.nodak.edu) is a web-based resource providing access a database of mathematically related PhD supervisors and their students that has been visualised in various ways (Horowitz *et al*. 2008). Turing's supervisor, the Princeton University mathematician Alonzo Church (1903–1995), has an entry that includes Turing (Mathematics Genealogy Project 2016).

Turing himself only had one PhD student, the mathematician Robin Gandy (1919–1995) at the University of Cambridge. Figure 12 provides a simplified PhD family tree for Alonzo Church. The middle branch includes another of Church's students, the mathematician Dana Scott, who while at Oxford University later supervised Jack Copeland, now a significant Turing scholar (Copeland 2012, Copeland *et al*. 2017). Robin Gandy was based at the Oxford Mathematical Institute contemporaneously as well.

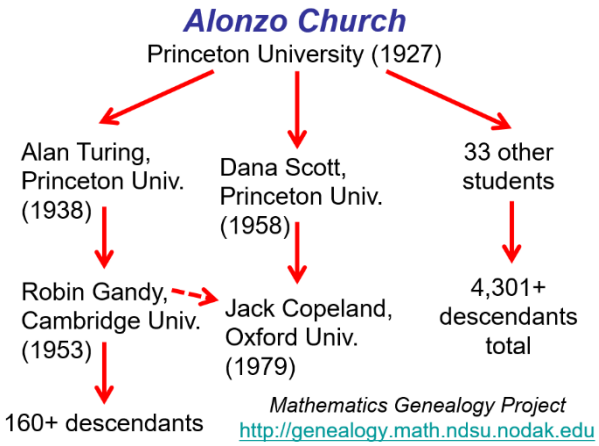

Figure 12: *PhD "family tree" for Alonzo Church, Turing's PhD supervisor at Princeton University (Mathematics Genealogy Project 2016)*

Online, Google Scholar (http://scholar.google.com) provides information about academic publications, including an entry for Turing (Google Scholar 2016), but without any significant visualisation facilities, apart from graphs of citations by year.

The Microsoft Academic Search website facility (http://academic.research.microsoft.com), produced through a project by Microsoft Research, is similar to Google Scholar, with a smaller corpus of publications, but better visualisation facilities. It can graphically present co-authors, citing authors, and a selection of transitive co-authorship links between any two authors (Bowen and Wilson 2012). Figure 13 shows a visualised display of some of Turing's top citing authors. More recently, Microsoft has relaunched the facility as Microsoft Academic (http://academic.microsoft.com), with a better corpus of publications but without the same visualisation facilities. E.g., for Turing's entry, see Microsoft Academic (2016).

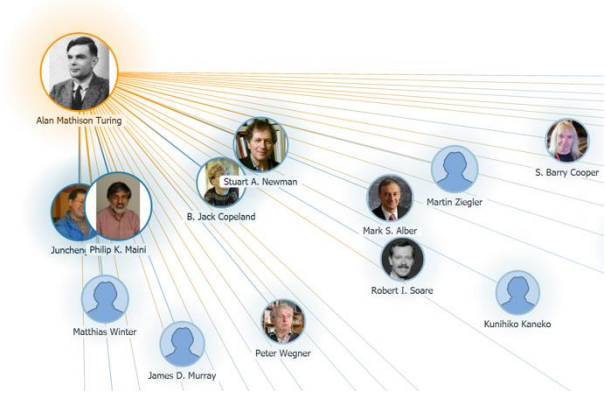

Figure 13: *Detail of citations graph for Turing (Microsoft Academic Search 2016)*

5. EPITAPHS AND CONCLUSION

Turing died in Wilmslow, Cheshire, on 7 June 1954, purported through suicide, although a modern investigation might have produced a more open verdict (Copeland 2012, pp. 223–234). Earlier that year, in a postcard to his friend and former PhD student, Robin Gandy, Turing had written a short poem (Copeland 2012, p. 161):

> Hyperboloids of wondrous Light;
> Rolling for aye through Space and Time;
> Harbour those Waves which somehow Might;
> Play out God's holy pantomime

– an appropriate epitaph for someone who studied Einstein's work in his teens. Geoffrey Jefferson, Professor of Neurosurgery at the University of Manchester, aptly described Turing as "a sort of scientific Shelley" (Bowen 2013). Both the poet Percy Bysshe Shelley (1792–1822) and Turing were rather maverick individualistic geniuses and both died before their time.

Figure 14: *Left: Turing bust donated by computer science students, Southwest University, Chongqing, China (Bowen 2016); right: and slate sculpture of Turing by Stephen Kettle (2012), Bletchley Park, UK (Photographs by J. P. Bowen, 2014)*

Turing is an important figure in the history of mathematics (Bowen 1996) but he has also been influential culturally in many fields, including art (Lomas 2016), film (IMDb 2014), literature (Beckett 2012), music (Rogers 2014), poetry (Clements 2016), sculpture (Kettle 2012, see also Figure 14), and theatre (IMDb 1996). Mathematics can have visually aesthetic qualities (Wolfram 2002, p. 11) and has sometimes stimulated artists (Grossman and Sebline 2015). Turing has certainly contributed to inspiring the arts in many ways, partly because of his multifaceted and short life.

Bernard Richards was Turing's last Master's student at the University of Manchester from 1953. In 2012, the year of Turing's centenary, Richards commented:

> "I would liken Turing to a four-faced pyramid. He had the skills that made him famous for code-breaking, he had his work in biology, he had his work in mathematics and his work in computer

> hardware and programming. All four sides of a pyramid come together to make a peak. That was Turing."
>
> "The day he died felt like driving through a tunnel and the lights being switched off."
>
> – Bernard Richards
> (Manchester Evening News 2012)

Despite the sadness surround Turing's death and his persecution by the British legal system as a homosexual, Alan Turing is now a revered figure, with great scientific and cultural influence, including in the arts. We can celebrate his significant contributions to science and his position in the pantheon of great 20th century figures is now assured.

6. ACKNOWLEDGEMENTS

Thank you to Alan Turing for his influence on mathematics in general and theoretical computer science in particular. An earlier talk with an abstract was presented at the EVA/Minerva Jerusalem conference in 2015 (Bowen 2015). In addition, thank you to Bernard Richards for permission to reproduce Figure 4 and Figure 5 from his thesis (Richards 1954) and to Richard Banach (University of Manchester) for scanning these. The author is grateful to Museophile Limited for financial support.

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